CHAPTER 23

TIME, DISTANCE and SPEED

You will often encounter problems on time and distance. Before we delve in to the short-cut methods, let us brush up some standard rules

We all know that Speed, Distance and Time are interrelated.

A. Speed = Distance/Time

B. Time = Distance/Speed

C. Distance = Speed × Time

In some questions you will be given speed in kilometres per hour (km/hr) and you will have to convert it in meters per second (m/s) and vice versa.

We know that 1 km equals 1000 meters and 1 hour is 3600 seconds.

$$1000/3600 = 5/18$$

So, when converting km/hr in to m/sec, use the formula

$$Z \text{ km/hr} = \left(z \times \frac{5}{18} \right) \text{m/sec}$$

And while converting m/sec in to km/hr, use the formula

$$Z \text{ m/sec} = \left(z \times \frac{18}{5} \right) \text{km/hr}$$

Now that we have learnt the basic formulas, we will solve two basic questions (just to brush up) before we move to the real challenging questions ahead.

Q1) A bus travels a distance of 400 km in 10 hours. What is its speed in km/hr?

Ans: Speed = Distance/Time → 400/10 = 40 km/hr.
The speed of the bus is 40 km/hr.

Q2) The distance between two stations is 720 km. A train takes 4 hours to cover this distance. Calculate the speed of the train in km/hr and m/s.

Ans: We know that Speed = Distance/Time = 720/4 = 180 km/hr. Next, to convert speed into m/s we have to multiply 180 by 5/18. So required speed = 180 × 5/18 = 50 m/s.
Now that we have brushed up the basics, we will see some advanced problems and direct methods of solving them quickly.

Q3) A man travels to work every day in his car. While going to office, he drives at a speed of 30 kmph and on the return journey he drives at a speed of 45 kmph. What is his average speed of travel?

Ans: 37.5 kmph (average of 30 and 45) is incorrect as the time travelled is different in both the cases and only the distances are same.
So, let the distance between the home and office be *y* kms

Therefore, the time taken on his onward journey = $y/30$ hours and time taken on his return journey = $y/45$ hours

So, the total time taken for the onward plus return journey = $y/30 + y/45 = 5y/90$ hours.

Also, the total distance travelled both ways = $y + y = 2y$ km

Therefore, the average speed = $\dfrac{2y}{\dfrac{5y}{90}}$

$$y = \frac{2y \times 90}{5y} = 36 \text{ km/hr}$$

From the above calculation, we get the average speed as 36 km/hr. Now let me share a short-cut method that can be used in such cases where different speeds are given.

If the distances are same and if you have to find the average speed, then the direct formula is

$$\text{Direct Formula Number 1} = \frac{2ab}{a+b}$$

(where 'a' and 'b' are the different speeds)

So, in the above example, the values of a and b are 30 and 45 respectively. Hence, our direct answer would be:

$$\frac{2 \times 30 \times 45}{30 + 45}$$

$$= \frac{2700}{75} = 36 \text{ km/hr}$$

Hence the average speed is 36 km/hr.

So you see we got the answer in a much quicker manner using the direct formula that I have shown here.

Q4) Two runners cover the same distance at the rate of 18 km/hr and 20 km/hr respectively. Find the distance travelled when one takes 24 minutes more than the other.

Ans: Let the distance be x kms.

Then the time taken by the first runner is $x/18$

The time taken by the second runner is $x/20$.

Now, $x/18 - x/20 = 24/60$

$$\frac{20x - 18x}{20 \times 18} = 24/60$$

$2x/360 = 24/60$

$x = 24/60 \times 360/2$

$x = 72$

Ans: The distance travelled is 72 kms.

We did get the answer that the distance is 72 kms. However, this method took a little time. Let me now give you another direct short-cut. This will save a few seconds and we all know that in competitive exams even a few seconds make a lot of difference.

Direct formula Number 2:

$$\text{Distance} = \frac{\text{Multiplication of Speeds}}{\text{Difference of Speeds}} \times \text{Difference in Time taken}$$

$$x = \frac{18 \times 20}{20 - 18} \times \frac{24}{60} \text{ and therefore } x = 72 \text{ kms.}$$

Yes, we get the same answer, viz., 72 kms!

Q5) A man sets out to cycle from Mumbai to Pune. At the very same time, his friend sets out to cycle from Pune to Mumbai (opposite direction). After crossing each other, they complete their journeys in 4 hours and 9 hours respectively. At what rate does the second man cycle if the first man is cycling at the rate of 12 km/hr?

Such questions are called 'Meeting Point' questions because the travellers are travelling from opposite points and meeting at a common place. The distance (Mumbai

to Pune) for both is constant. You will often encounter such questions in competitive exams.

Let me give you an amazing direct formula to solve such questions.

Let us say the cyclists names were M and N.

Our **Direct Formula Number 3** says:

$$\frac{\text{Speed of M}}{\text{Speed of N}} = \frac{\text{Square Root of Balance Time of N}}{\text{Square Root of Balance Time of M}}$$

$$\frac{12}{\text{Speed of N}} = \frac{4}{9}$$

$$\frac{12}{\text{Speed of N}} = \frac{2}{3}$$

Speed of N = 18 km/hr

Please note that each of these direct formulae are mathematically accurate. I am not providing here the derivation of these formulas as it is beyond the scope of this book.

Q6) A doctor covers a certain distance in an Uber cab. Had the cab driver driven 5 km/hr faster, the doctor would have taken 1 hour less to reach. Had the cab driven 3 km/hr slower, the doctor would have taken an hour more to reach. Find the original speed and distance.

I have a secret formula to solve such type of questions. But before I show you that formula, let us solve this question by the conventional method.

Traditional Method: Let the distance be D and the initial speed be *x* km/hr

We know that Time = Distance/Speed

Regular Time = D/*x*

Faster Time = D/*x*+5 (Speed increased by +5)

Slower Time = D/x–3 (Speed reduced by –3)

We know that when the cab drives 5 km/hr faster, the time saved is 1 hour.

Faster time is regular time less one hour

$$\frac{D}{x+5}=\frac{D}{x}-1 \quad \text{(Let this be Equation 1)}$$

We also know that when the cab drives 3 km/hr slower, the time taken is an additional 1 hour

Slower time is regular time plus one hour

$$\frac{D}{x-3}=\frac{D}{x}+1 \text{ (Let this be Equation 2)}$$

Let us simplify the equations

Equation 1

$$\frac{D}{x+5}=\frac{D}{x}=-1$$

$$=\frac{D}{x}-\frac{D}{x+5}=1$$

$$=\frac{Dx+5D-Dx}{x(x+5)}=1$$

$$=\frac{5D}{x(x+5)}=1$$

Equation 2

$$\frac{D}{x-3}=\frac{D}{x}+1$$

$$=\frac{D}{x-3}-\frac{D}{x}=1$$

$$=\frac{Dx+3D-Dx}{x(x-3)}=1$$

$$=\frac{3D}{x(x-3)}=1$$

Since both equations equal to 1, we know that

$$\frac{5D}{x(x+5)}=\frac{3D}{x(x-3)}$$

$5(x-3)=3(x-5)$

$5x-15=3x+15$

$2x=30$ and hence $x=15$

So, the original speed is 15 km/hr.

Now that we have got the original speed, let us calculate the distance by substitution. Look at the equation 1 above:

$$\frac{D}{x}-\frac{D}{x+5}=1$$

$$=\frac{D}{15}-\frac{D}{15+5}=1$$

$$=\frac{D}{15}-\frac{D}{20}=1$$

$$=\frac{20D-15D}{15\times 20}=1$$

$$=\frac{5D}{300}=1 \text{ and hence } D = 60 \text{ kms}$$

So, the original speed is 15 km/hr and the distance is 60 kms.

Phew! That was a long calculation. Now let me present to you a short-cut.

This short-cut can **only** be used when the time increased and decreased is the same (+1 hour and -1 hour in this case)

Direct Method to Find the Speed:

$$\text{Speed}=\frac{2\times \text{Increase in Speed}\times \text{Decrease in Speed}}{\text{Difference in Increase and Decrease in Speeds}}$$

Our increase in Speed is 5 and decrease in Speed is 3

Using this amazing equation, we have

$$\text{Speed}=\frac{2\times 5\times 3}{5-3}=\frac{30}{2}=\ 15 \text{ km/hr}$$

Now that we have got the speed, we can substitute and find the distance. However, we again have a special formula for finding the distance. Please refer to the

Direct Formula No 2 given in Question 4 in this chapter

Direct Formula Number 2:

$$\text{Distance} = \frac{\text{Multiplication of Speeds}}{\text{Difference of Speeds}} \times \text{Difference in Time taken}$$

$$\text{Distance} = \frac{(15+5) \times (15-3)}{(15+5)-(15-3)} \times 5-3$$

$$\text{Distance} = \frac{20 \times 12}{20-12} \times 2$$

$$\text{Distance} = \frac{240}{8} \times 2$$

Distance = 60 kms.

So we know that the speed is 15 km/hr and the distance is 60 kms.

I hope you liked the amazing short-cuts and direct methods that I have given to you in this chapter. Please use them to your advantage!